Exponential inequalities

For the exponential function is:

1) for $a > 1$ is $a^{f(x)} > a^{g(x)} \Leftrightarrow f(x) > g(x)$

2) for $0 < a < 1$ is $a^{f(x)} > a^{g(x)} \Leftrightarrow f(x) < g(x)$

Examples:

1) Solve the inequalities:

a) $5^{-7x+3} > 5^{-3}$

b) $0{,}35^{x-1} < 0{,}35^{2x+2}$

c) $2^{x^2-3} > 2$

d) $2^x < 7^x$

Solutions:

a) $5^{-7x+3} > 5^{-3} \rightarrow$ as the basis is 5, the direction of inequality does not change

$$-7x+3 > -3$$
$$-7x > -3-3$$
$$-7x > -6$$
$$x < \frac{6}{7}$$

b) $0{,}35^{x-1} < 0{,}35^{2x+2} \rightarrow$ the basis is 0,3 ; $0 < 0{,}3 < 1$,we must face the direction of inequality

$$x-1 > 2x+2$$
$$x-2x > 2+1$$
$$-x > 3$$
$$x < -3$$

c) $2^{x^2-3} > 2$

$2^{x^2-3} > 2^1$

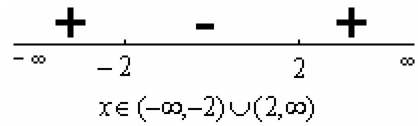
$x^2 - 3 > 1$

$x^2 - 4 > 0$

$x_{1,2} = \dfrac{-0 \pm 2}{2}$

$x_1 = 2$

$x_2 = -2$



d) $2^x < 7^x$

$\dfrac{2^x}{7^x} < 1$

$\left(\dfrac{2}{7}\right)^x < 1$

$\left(\dfrac{2}{7}\right)^x < \left(\dfrac{2}{7}\right)^o$ $\rightarrow$ the basis is between 0 and 1 , so, we must face the direction of inequality

x >0

2) Solve the inequalities:

a) $5^{2x+1} > 5^x + 4$

b) $25^x < 6 \cdot 5^x - 5$

c) $\sqrt{9^x - 3^{x+2}} > 3^x - 9$

Solutions:

a) $5^{2x+1} > 5^x + 4$

$5^{2x} \cdot 5^1 - 5^x - 4 > 0 \rightarrow$ replacement $5^x = t$

$t^2 \cdot 5 - t - 4 > 0$

$t_{1,2} = \dfrac{1 \pm 9}{10}$

$t_1 = 1$

$t_2 = -\dfrac{4}{5}$

$t \in (-\infty, -\dfrac{4}{5}) \cup (1, \infty)$

go back in replacement $5^x = t$

$5^x = -\frac{4}{5}$ no solution or $5^x = 1$

$5^x = 1$

$x = 0 \rightarrow x \in (0, \infty)$

Now the interval $t \in (1, \infty)$ $\longrightarrow$ transformed into $x \in (0, \infty)$

$x \in (0, \infty)$ is solution!

b) $25^x < 6 \cdot 5^x - 5$

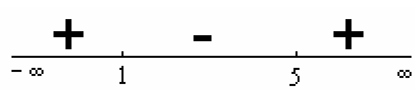
$5^{2x} - 6 \cdot 5^x + 5 < 0 \rightarrow$ replacement $5^x = t$

$t^2 - 6t + 5 < 0$

$t_{1,2} = \frac{6 \pm 4}{2}$

$t_1 = 5$

$t_2 = 1$



$t \in (1,5)$, go back in replacement $5^x = t$

$5^x = 1$, $x = 0$ or $5^x = 1$, $x = 1$

$x \in (0,1)$ is the final solution.

c) $\sqrt{9^x - 3^x \cdot 3^2} > 3^x - 9$

$\sqrt{3^{2x} - 3^x \cdot 9} > 3^x - 9 \rightarrow$ replacement $3^x = t$

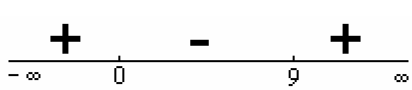
$\sqrt{t^2 - 9t} > t - 9$

$\left[t^2 - 9t \geq 0 \wedge t - 9 < 0\right]$ $\vee$ $\left[t^2 - 9t \geq (t-9)^2 \wedge t - 9 \geq 0\right]$

$t_{1,2} = \frac{9 \pm 9}{2}$

$t_1 = 0$ $t < 9$

$t_2 = 9$



$t \in (-\infty, 0] \cup [9, \infty)$

$t \in (-\infty, 0]$

this interval "does not work" because it is $3^x = t$

$t^2 - 9t > t^2 - 18t + 81$

$-9t + 18t > 81$ $t \geq 9$

$9t > 81$

$t > 9$

So: $t > 9$

$3^x > 9$

$3^x > 3^2$

$x > 2$

the final solution